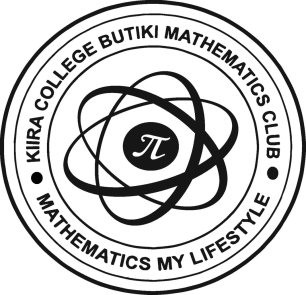
***KIIRA COLLEGE BUTIKI MATHEMATICS CLUB  
  
MATHLETICS CONTEST 2019  
  
SENIOR MATHLETES CATEGORY SUGGESTED SOLUTIONS***

***Section A:  
Qn.1.*** ***Solution 1:*** Let . Since ; squaring both equations and adding the results gives , so . The maximum value of is , hence and .  
***Solution 2:*** The value of the expression is maximum when . So will become then is , giving . Note that (since ), then the required value is .

***Qn.2.*** Because the mean of the first terms is , their sum is . Therefore, the term is , and the term is .

***Qn.3.*** We are given that . Substituting in the values of the expression, we have . Notice that the last term is equal to , hence the whole expression terminates into .

***Qn.4.*** Checking other sizes, we have:  
One square = 1 square in total.  
Four squares, one square = 5 squares in total.

Nine squares, four   
squares, one square = 14   
squares in total.

Sixteen squares, nine   
 squares, four   
squares, one square in total.  
We observe that for square, there is 1 square in total, equal to ; for square, the squares are 5, equal to ; for square, they are 14, equal to ; then for square, the squares are 30, equal to and so on. Hence, for square, there are squares.

***Qn.5.*** The least possible value of is 2, hence has possible values of and as 0 and 1. Note that doesn’t work out, as well as . Hence, . If , the possible values of and are 0, 1, and 2. Notice still that cannot be 0, so we try values or . The latter is in base 3; the former is , which works. Hence, by trial and error, the values of and are 1, 2, 0 and 3 respectively.

***Qn.6.*** Let , and let and be and respectively, so that we have . Cubing both sides, we get . Note that ; so . Substituting in for and ; . Note that is a factor, so we factorize this as . The quadratic has no solutions hence we have that giving . Therefore, the difference:  
 .  
  
***Qn.7.*** If . Note that . So .

***Qn.8.*** We are given that slope . From the general equation of a straight line , if follows that (by hypothesis); ; setting . Since when , then the required point is .

***Qn.9.*** Let and ; note that , hence . The required value is . Since and , the required value is .

***Qn.10.*** Notice that , so . Since ; .

***Section B:  
Qn.11.*** Subtracting the given equations, we have that giving . However, none of these values satisfies the equations. Note that substituting in each of the equations gives for and for ; giving pairs . Substituting in . The quadratic has no real solutions (roots) hence, ; giving a pair . Therefore, the required pairs are and .

***Qn.12.*** ; note that for any real positive number , is always equal to , hence our function simplifies to . Substituting in accordingly, our expression becomes which we rearrange as which terminates into giving us as the solution.

***Qn.13.*** The word “MATHLETICS” appears every tenth row, i.e. 1, 11, 21, 31 etc. while the word “CONTEST” appears every seventh row i.e. 1, 8, 15, 22, etc. Using the Least Common Multiple of 10 and 7 i.e. 70, it can be seen that both words appear every 70 rows i.e. in the first and only the first row of every set of 70 rows i.e. 1, 71, etc. So, .

***Qn.14.*** Let denote a used space and denote a vacant space. The problem is the same as finding the probability , that in a string of and there are at least 2 consecutive . Then , is the probability that no two are consecutive. In a string of , there are 13 spaces in which to insert to create a string in which no two are consecutive. Thus; since the sample .

***Qn.15.*** From the question, we know that Mrs. Em’s children’s ages are three numbers whose product is 90, and their sum is on the door. These numbers could be one of these triples; (1,1,9),(1,2,45),(1,3,30),(1,5,18),(1,6,15), (1,9,10), (2,3,15),(2,5,9),(3,3,10),(3,5,6). Now, the census taker did not have enough information when she was given the sum, implying that for the given sum, there was more than one triple. Note that out of all the triples above, only two pairs of them share a sum, i.e. (1,9,10),(2,3,5) which have a sum of 20, and (2,5,9),(3,3,10) with a sum of 16. But all Mrs. Em’s children are aged above 1 year: this eliminates the triple (1,9,10). Now if the number on the door was 20, the census taker could have known the ages of the children straightaway since one triple out of the two whose sum is 20 is already invalid; but she did not. It follows then that the number on the door was 16. Now the census taker was satisfied when she knew there was a youngest child. So, from the two triples (2,5,9) and (3,3,10), the one in full agreement with this condition of the youngest child is the former, hence, the ages of the children are 2, 5 and 9 .

***Bonus:  
1:*** Let be a positive integer. Prove that: .  
***2:*** Let be the product of the positive integers from 1 to 2019, ignoring the multiples of 3. What is the remainder when is divided by 3?